Systemic Risk
in a Structural Model of
Bank Default Linkages

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Structural Model of Default

• Description:
  – Individual bank defaults iff assets < debt
  – Introduce default correlation through asset correlation
  – Study a banking system with N individual banks

• Analogous to default risk in a bank portfolio but there are two crucial differences:
  – Typically use “small” correlation in bank portfolios, but we empirically find large correlations in the banking system
  – Typically assume “large/infinite” number N but actual number is small in the banking system
Micro-prudential Regulation and Banking Sector Default

- Micro-prudential regulation addresses individual default probability $p_i = p$ for $i = 1, \ldots, N$ banks
- To describe banking sector default, define
  - Indicator variable for default of bank $i$: $X_i$
  - Default frequency: $M_N = \frac{\sum_{i=1}^{N} X_i}{N}$
- **If** asset correlation=0, Law of Large Numbers implies
  
  $M_N \rightarrow p$
  
  - suggests that default frequency is “close to” individual default probability,
- Focus on micro-prudential regulation
Density of Default Frequency $M_N$
(Correlation $\rho=0$; N=1,000 Banks)

For correlation=0: Here $0.3146\%$

For correlation=0: $M_N \sim N(p, \sigma_N)$, $\sigma_N = \sqrt{\frac{p(1-p)}{N}}$; Here $\sigma_N = 0.3146\%$

Parameter: ind. def. prob. $p=1\%$
Density of Default Frequency $M_N$
(Correlation $\rho=63.8\%$; $N=1,000$ Banks)

Parameter: ind. def. prob. $p=1\%$
Systemic Risk Measure

• Summary of observations from previous slides:
  – Actual numbers N are “large” but too small to adequately capture “infinity”
  – In addition, correlations are far from zero, in particular close to 1 (maximum)
• Default frequency “spreads” out to the right.
• Default frequency larger than micro-prudential reference level \( p = E[M_N] \) is problematic
  – Define Conditional Expected default frequency:
  – Systemic risk measure
  – Foundation for macro-prudential regulation
Systemic is Sizeable and Depends Non-linearly on Correlation

Parameter: ind. def. prob. p=1%

- $N=10$
- $N=20$
- $N=1000$

Min $\rho$ 75%
Average $\rho$ 82%
Max $\rho$ 92%
Evolution of Our Systemic Risk Measure
Conclusion

• Approach to systemic risk based on well-known structural model of credit risk

• Asset correlation
  – Strong non-linear impact on systemic risk measure
  – Empirically, increasing over time and typically “large”
  – Strong increases may signal systemic stress

• “Large” correlations mean macro-prudential regulation required